Please check the examination details below before entering your candidate information

Candidate surname	Other names
Solved by Mr. Pabitra	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 9 January 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

Solved by Mr. Pabitra | Math Tonic

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) \mathrm{d}x$$

giving your answer in simplest form.

(4)

Write as powers: $8x^3 - 6x^{1/2} - \frac{2}{5}x^{-3}$

$\int = \frac{8x^4}{4} - 6 \times \frac{x^{3/2}}{\frac{3}{2}} - \frac{2}{5} \times \frac{x^{-2}}{-2} + c$

$= 2x^4 - 4x^{3/2} + \frac{1}{5}x^{-2} + c$

MATH TONIC

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $\left(-2\sqrt{3}, 5\right)$
- the point B has coordinates $\left(7\sqrt{3}, 8\right)$
- the straight line l_1 passes through A and B

(a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A.

(b) Find the equation of l_2, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(a) Gradient $= \frac{8-5}{7\sqrt{3}-(-2\sqrt{3})} = \frac{3}{9\sqrt{3}} = \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{9}$

so the gradient is $p\sqrt{3}$ with $p = \frac{1}{9}$

(b) Gradient of $l_2 = -1 \div \frac{\sqrt{3}}{9} = -\frac{9}{\sqrt{3}} = -3\sqrt{3}$

$y - 5 = -3\sqrt{3}(x + 2\sqrt{3})$ $\quad y = -3\sqrt{3}x - 18 + 5$

$\boxed{y = -3\sqrt{3}x - 13}$

MATH TONIC

Question 2 continued

(Total for Question 2 is 5 marks)

3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

(a) P is in thousands: $t = 5, P = 58$; $t = 10, P = 65$

$58^2 = a + 125b$ so $3364 = a + 125b$

$65^2 = a + 1000b$ so $4225 = a + 1000b$

Subtract: $875b = 861$, $b = 0.984$

$a = 3364 - 125 \times 0.984 = 3364 - 123 = 3241$

$\boxed{a = 3241, \quad b = 0.984}$

(b) $P = 85$: $85^2 = 3241 + 0.984T^3$

$7225 - 3241 = 0.984T^3$ $\quad T^3 = \frac{3984}{0.984} = 4048.78...$

$T = \sqrt[3]{4048.78...} = 15.938...$

$\boxed{T = 15.9}$

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 5 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \qquad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1}

(1)

(b) $\dfrac{5}{\left(3a^{1-x}\right)^{-2}}$

(3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$

(2)

(b) Hence solve

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

(3)

(i)(a) $a^{3x+1} = (a^x)^3 \times a$

$\boxed{a^{3x+1} = ay^3}$

(i)(b) $\dfrac{5}{(3a^{1-x})^{-2}} = 5(3a^{1-x})^2 = 5 \times 9 \times a^{2-2x}$

$= 45 \times a^2 \times (a^x)^{-2} = 45a^2y^{-2}$

$\boxed{= \dfrac{45a^2}{y^2}}$

(ii)(a) $3^{4t+2} = 3^2 \times 3^{4t} = 9 \times (9^t)^2 = 9p^2$

$3(9p^2 + 1) = 82p \qquad 27p^2 + 3 = 82p$

$27p^2 - 82p + 3 = 0 \qquad$ as required

(ii)(b) $(27p - 1)(p - 3) = 0 \qquad p = \dfrac{1}{27}$ or $p = 3$

$9^t = 3 = 9^{1/2}$: $t = \dfrac{1}{2} \qquad 9^t = \dfrac{1}{27}$: $3^{2t} = 3^{-3}$, $t = -\dfrac{3}{2}$

MATH TONIC

Question 4 continued

$$t = \frac{1}{2} \quad \text{or} \quad t = -\frac{3}{2}$$

MATH TONIC

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

(a) $y = 4x^3 + 2x^{-1} + 9$

$\frac{dy}{dx} = 12x^2 - 2x^{-2}$

(b) The tangent $y = k - 5x$ has gradient -5:

$12x^2 - \frac{2}{x^2} = -5 \qquad (\times x^2)$

$12x^4 + 5x^2 - 2 = 0 \qquad$ as required

(c) $(4x^2 - 1)(3x^2 + 2) = 0$

$x^2 = \frac{1}{4}$ $(3x^2 + 2 = 0$ has no real roots), $x > 0$ so $x = \frac{1}{2}$

$y = 4 \times \frac{1}{8} + 2 \div \frac{1}{2} + 9 = \frac{1}{2} + 4 + 9 = 13.5$

P lies on the tangent: $13.5 = k - 5 \times \frac{1}{2} = k - 2.5$

$k = 16$

MATH TONIC

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $f'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $f(x)$

(7)

Curve C is transformed to the curve with equation $y = f(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

(a) $f'(4) = 7$: $\dfrac{2 \times 4^2 + 4a + b}{4\sqrt{4}} = 7$

$32 + 4a + b = 56$ $\quad 4a + b = 24$ as required

(b) $4a + b = 24$ and $a + b = -9$: $3a = 33$, $a = 11$, $b = -20$

$f'(x) = \dfrac{2x^2 + 11x - 20}{4x^{1/2}} = \dfrac{1}{2}x^{3/2} + \dfrac{11}{4}x^{1/2} - 5x^{-1/2}$

$f(x) = \dfrac{1}{2} \times \dfrac{2}{5}x^{5/2} + \dfrac{11}{4} \times \dfrac{2}{3}x^{3/2} - 5 \times 2x^{1/2} + c$

$= \dfrac{1}{5}x^{5/2} + \dfrac{11}{6}x^{3/2} - 10x^{1/2} + c$

$f(4) = -5$: $\dfrac{32}{5} + \dfrac{88}{6} - 20 + c = -5$ $\quad \dfrac{16}{15} + c = -5$, $\quad c = -\dfrac{91}{15}$

$$\boxed{f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} - \frac{91}{15}}$$

(c) $f(x - 3)$ is a translation 3 to the right:

Question 6 continued

$Q = (7, -5)$

MATH TONIC

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C.

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k.

(3)

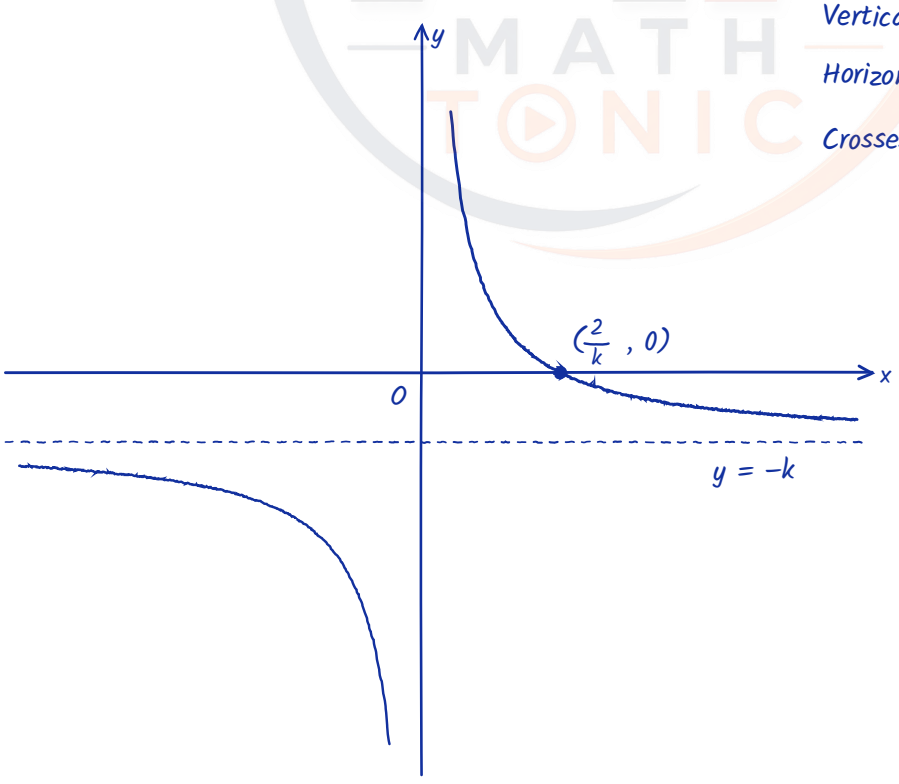
The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k.

(5)

(a) Sketch of $y = \frac{2}{x} - k$



Vertical asymptote $x = 0$

Horizontal asymptote $y = -k$

Crosses x-axis: $\frac{2}{x} = k$, $x = \frac{2}{k}$

Question 7 continued

(b) $-kx - 6 = \frac{2}{x} - k \qquad (\times x)$

$-kx^2 - 6x = 2 - kx \qquad kx^2 + (6 - k)x + 2 = 0$

2 distinct points: $b^2 - 4ac > 0$

$(6 - k)^2 - 4 \times k \times 2 > 0 \qquad 36 - 12k + k^2 - 8k > 0$

$k^2 - 20k + 36 > 0 \qquad (k - 2)(k - 18) > 0$

Critical values 2 and 18; outside them, and k is positive:

$\boxed{0 < k < 2 \quad \text{or} \quad k > 18}$

MATH TONIC

Question 7 continued

MATH TONIC

Question 7 continued

(Total for Question 7 is 8 marks)

8.

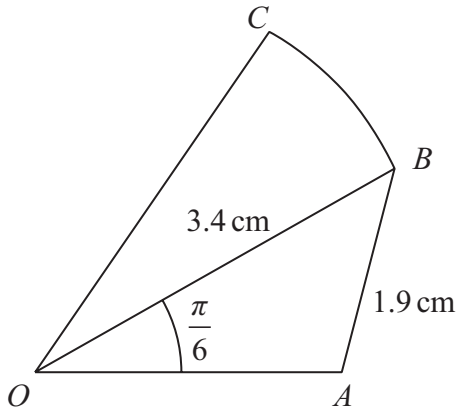


Figure 1

Figure 1 shows a sketch of a design for a badge.
The design consists of a triangle OAB joined to a sector OBC of a circle with centre O
In the design

- $OB = 3.4\,\text{cm}$
- $AB = 1.9\,\text{cm}$
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB, giving your answer in radians to 4 significant figures,

(3)

(b) find the area of triangle OAB, in cm^2, giving your answer to 3 significant figures.

(2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is $3:2$

(c) show that angle BOC is 0.462 radians to 3 significant figures.

(3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer.

(5)

(a) Sine rule: $\frac{\sin OAB}{3.4} = \frac{\sin \frac{\pi}{6}}{1.9}$ $\sin OAB = \frac{3.4 \times 0.5}{1.9} = 0.89473...$

$OAB > \frac{\pi}{2}$ so it is obtuse: $OAB = \pi - 1.1078... = 2.0337...$

angle $OAB = 2.034$ rad

(b) $OBA = \pi - \frac{\pi}{6} - 2.0337... = 0.58425...$

Question 8 continued

Area $= \frac{1}{2} \times 3.4 \times 1.9 \times \sin(0.58425...) = 1.7815...$

Area of OAB $= 1.78\ cm^2$

(c) Sector OBC $= \frac{3}{2} \times 1.7815... = 2.6723...$

$\frac{1}{2} \times 3.4^2 \times \theta = 2.6723...$ $\theta = \frac{2.6723...}{5.78} = 0.46234...$

angle BOC $= 0.462$ (3 s.f.) as required

(d) $OA^2 = 3.4^2 + 1.9^2 - 2 \times 3.4 \times 1.9 \cos(0.58425...) = 4.393...$, $OA = 2.0959...$

Arc BC $= 3.4 \times 0.46234... = 1.5719...$

Perimeter $= OA + AB + \text{arc } BC + CO = 2.0959... + 1.9 + 1.5719... + 3.4 = 8.967...$

Perimeter $= 9$ cm

MATH TONIC

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

9.

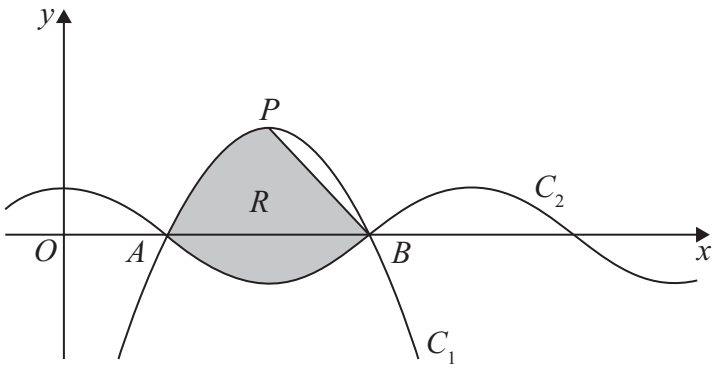


Figure 2

(a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a, b and c are constants to be found.

(3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

(b) state the coordinates of P

(2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x-axis at point A and at point B, as shown in Figure 2,

(c) (i) state the x coordinate of B

(ii) state the value of k

(iii) state the period of C_2

(3)

The line segment L joins P and B.
The region R, shown shaded in Figure 2, is bounded by L, C_1 and C_2

(d) Use inequalities to define R.

(5)

(a) $6x - \frac{27}{4} - x^2 = -(x^2 - 6x) - \frac{27}{4} = -[(x-3)^2 - 9] - \frac{27}{4}$

$= \frac{9}{4} - (x-3)^2$

Question 9 continued

(b) Maximum when $x = 3$:

$$\boxed{P = \left(3, \frac{9}{4}\right)}$$

(c)(i) $C_1 = 0$: $(x-3)^2 = \frac{9}{4}$, $x = 3 \pm \frac{3}{2}$ A: $x = \frac{3}{2}$, B: $x = \frac{9}{2}$

$$\boxed{x \text{ coordinate of } B = \frac{9}{2}}$$

(c)(ii) A is the first zero of $\cos(kx)$: $k \times \frac{3}{2} = \frac{\pi}{2}$

$$\boxed{k = \frac{\pi}{3}}$$

(c)(iii) Period $= \frac{2\pi}{k} = 2\pi \div \frac{\pi}{3}$

$$\boxed{\text{Period} = 6}$$

(d) Line L through $P\left(3, \frac{9}{4}\right)$ and $B\left(\frac{9}{2}, 0\right)$: gradient $= \left(0 - \frac{9}{4}\right) \div \frac{3}{2} = -\frac{3}{2}$

L: $y = -\frac{3}{2}\left(x - \frac{9}{2}\right) = -\frac{3}{2}x + \frac{27}{4}$

R is above C_2, below C_1 and below L:

$$\boxed{\cos\left(\frac{\pi}{3}x\right) \leq y \leq 6x - \frac{27}{4} - x^2, \quad y \leq -\frac{3}{2}x + \frac{27}{4}}$$

MATH TONIC

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS